The New Keynesian Model Part 1

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Macroeconomics II

Idea

- The real business cycle model has (almost) no role for nominal, money, shocks to matter.

- The model is inconsistent with some movements in macroeconomic aggregates.

- Introducing “bells and whistles” into the model may overcome some of these shortcomings but...
  - we have evidence that money matters for real variables.
  - maybe giving up the neutrality of money is more convincing than additional frictions/shocks.
  - Inflation is procyclical.

Idea II

- The New Keynesian model introduces non-neutrality of money.
- The non-neutrality arises from price-stickiness, i.e., firms do not always adjust their prices.
- Monetary policy shocks become an independent source for economic fluctuations.
- We will see that with sticky prices, economic fluctuations are no-longer efficient.
- Central banks not only can but also should affect economic fluctuations.

Empirical support for the non-neutrality of money

Outlook

We are going to see two arguments:

1. In the data, changes in the money supply change real variables, such as output, at the business cycle frequency.

2. Firms change their prices infrequently suggesting some price stickiness.

Money supply and monetary policy

- We want to determine the response of real variables to changes in the money supply.
- The money supply depends on the financial system and on the monetary base. The latter is set by the central bank.
- We begin by assuming that the central bank controls the money supply.
- Afterward, we assume that it controls the interest rate.

Monetary policy tools

The central bank has, broadly speaking, three sets of instruments:

1. Open market operations.
2. Reserve requirements.
3. Interests on reserves, discount rates, and REPOs.

Open market operations

- Before the great recession, this was the primary policy tool.
- These are purchases or sales of government bonds by the Federal Reserve.
- If the central bank buys bonds from the public, it pays with new currency (or central bank reserves), increasing the money supply.

Open market operations II

- Banks are obliged to hold a fraction of their deposits as reserves at the central bank.
- These used to pay no interest and, hence, banks had no incentives to hold excess reserves.
- When their reserves are low, they borrow liquidity from other banks overnight.
- The resulting overnight rate is called the FED Funds Rate.
- The central bank adjusts the amount of reserves by buying and selling bonds and, thereby, changes the Fed Funds Rate.

Reserve requirements

- Central bank regulations that require banks to hold a minimum reserve-deposit ratio.
- The central bank can change these requirements and, thereby, affect the demand for reserves.
- In practice, reserve requirements are not changed frequently.

Central banks since the Great Recession

- During the Great Recession, central banks increased reserves so much that all banks had excess reserves.
- The interest rate became zero.
- As a result, central banks started to use new tools to affect the quantity of money in the economy.

Interest rate on reserves

- To manage the demand for reserves, central banks have started to pay interest on reserves.
- The central bank is free to choose any interest rate.
- It can be positive, FED, or negative, ECB.

Discount rate

- The central bank can also directly lend money to banks through the so called discount window.
- These can also be long-term loans as the ECB has done.
- Again, the central bank is free to choose the interest rate. The ECB has also used negative interest rates.
- However, the interest rate is naturally above the interest rate on reserves.

Repurchase agreements (REPOs)

- The central bank may also wish to lend money to non-commercial banks.
- This is done through so called REPOs.
- Here, a private institution receives money and provides the central bank with a collateral (often bonds).
- The private institution promises to repurchase the collateral at a future point in time (often the next morning).
- For this, the central bank receives an interest at a pre-announced rate.

Monetary policy shocks

- We want to determine the response of real variables to changes in the monetary policy.
- We are going to simplify the policy set of the central bank and only look at changes in the money supply or the Fed Funds Rate.
- The problem is that monetary policy is endogenous. When the central bank expects a recession, it will increase the money supply (decrease the interest rate).
- This does not mean that the policy change has caused the recession!

Identifying monetary policy shocks

To identify how monetary policy changes affect real variables in the data, the literature tries to identify monetary policy shocks, that is, surprising changes in the monetary policy. Let S_t be the instrument of the monetary policy. The idea is to decompose this into a predictable and a surprise component:

$$S_t = f(\Omega_t) + \epsilon_t, \tag{1}$$

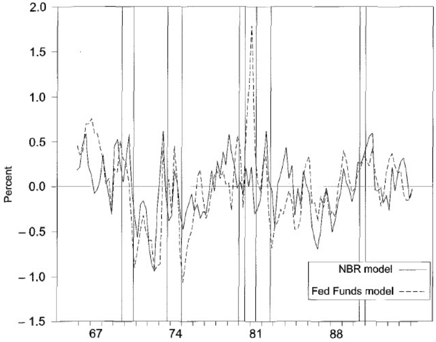
where f is some function to be estimated and Ω_t is the relevant information set of the central bank in period t. ϵ_t are the shocks to monetary policy possibly reflecting

- shocks to the preferences of the central bank.
- shocks to private-sector expectations that the central bank wants to accommodate.
- measurement error in real-time data.

Identifying monetary policy shocks II

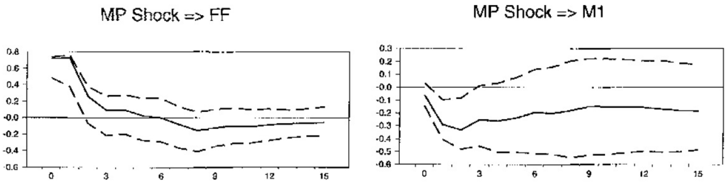
- The literature usually uses vector autogeressive models to identify the response of macroeconomic aggregates on monetary policy shocks.
- This usually requires some structural assumptions about the timing of events.
- This allows to estimate impulse responses of macroeconomic aggregates.
- Here, we are not going into these models.
- We simply look at the results from Christiano et al. (1999).

Time series of a monetary policy shock



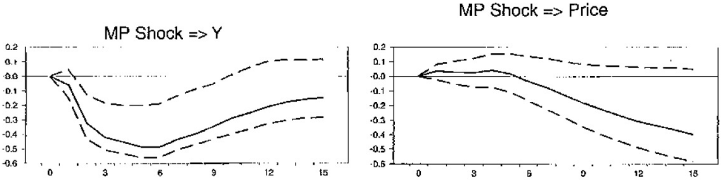
- Using the Fed Funds Rate or a monetary aggregate as policy instrument yields similar results.
- Several recessions are associated with too high interest rates.

The effects of a monetary policy shock



- We observe a persistent increase in the Fed Funds Rate for about 3 quarters.
- The money supply, measured as M1, falls.

The effects of a monetary policy shock II



- Output falls persistently with a peak response after about 5 quarters.
- Prices might fall but the effect is statistically insignificant.

How to make money matter

- The RBC model is inconsistent with these data facts. In that model, changes to the money supply (the nominal interest rate) have almost no effect on real variables.
- The reason is that prices are flexible. When the money supply increases, prices simply increase leaving the real money supply almost unchanged even in the short-run.
- We are going to see now that firms adjust prices only infrequently.
- When prices are sticky, changes in the money supply will affect real variables.

Price stickiness in the data

	Statistics	Euro area	US
CPI*	Frequency	15.1	24.8
	Average duration (*months*)	13.0	6.7
	Median duration (*months*)	10.6	4.6
PPI†	Frequency	20.0	n.a
Surveys‡	Frequency	15.9	20.8
	Average duration (*months*)	10.8	8.3
NKPC§	Average durations (*months*)	13.5–19.2	7.2–8.4
Internet prices¶	Frequency	79.2	64.3

- Alvarez et al. (2006) compare price stickiness in the Euro area to the US.
- In both regions, firms change prices infrequently.
- Price stickiness is yet more pronounced in Europe.

Calvo pricing with a money growth rule

Outlook

We are going to start with

- The central bank follows, as before, a money growth rule.
- The household derives utility from money.
- There is no physical capital.
- The household trades a bond with itself.
- Prices are sticky: The firms' ability to adjust their prices is stochastic.
- For price stickiness to be feasible, firms must be price-setters. Hence, we will use the framework of imperfect competition.

The household problem

$$\max_{C_t, H_t, B_{t+1}, M_{t+1}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \phi \frac{H_t^{1+\eta}}{1+\eta} + \varphi \ln \left(\frac{M_{t+1}}{P_t} \right) \right) \right\} \quad (2)$$

s.t.

$$C_t + \frac{B_{t+1}}{P_t} + \frac{M_{t+1} - M_t}{P_t} = \frac{W_t}{P_t} H_t + (1 + i_{t-1}) \frac{B_t}{P_t} + \Pi_t - T_t. \quad (3)$$

First order conditions

The first order conditions are:

$$\frac{\partial \Lambda_t}{\partial C_t} : C_t^{-\gamma} = \lambda_t \tag{4}$$

$$\frac{\partial \Lambda_t}{\partial B_{t+1}} : \beta^t \lambda_t = \mathbb{E}_t \Big\{ \beta^{t+1} \lambda_{t+1} (1 + i_t) \frac{P_t}{P_{t+1}} \Big\} \tag{5}$$

$$\frac{\partial \Lambda_t}{\partial M_{t+1}} : \beta^t \frac{\varphi}{P_t} \left(\frac{M_{t+1}}{P_t} \right)^{-1} + \beta^{t+1} \mathbb{E}_t \frac{\lambda_{t+1}}{P_{t+1}} = \beta^t \frac{\lambda_t}{P_t} \tag{6}$$

$$\frac{\partial \Lambda_t}{\partial H_t} : \phi H_t^{\eta} = \lambda_t \frac{W_t}{P_t} \tag{7}$$

Combining terms

Bond optimality:

$$C_t^{-\gamma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\gamma} (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \tag{8}$$

Money optimality:

$$\varphi \left(\frac{M_{t+1}}{P_t} \right)^{-1} = C_t^{-\gamma} \frac{i_t}{1 + i_t} \tag{9}$$

Hours optimality:

$$\phi H_t^{\eta} = C_t^{-\gamma} \frac{W_t}{P_t} \tag{10}$$

Final goods producer

The final goods producer uses all available intermediate input goods $y_{j,t}$ and bundles them to the (real) final output good using as production technology:

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} dj \right)^{\frac{\mu}{\mu-1}} . \tag{11}$$

The final goods producer takes the intermediate goods prices, $p_{j,t}$, as given and sells the final good at price P_t. It chooses inputs to maximize profits:

$$\max_{y_{j,t}} \left\{ P_t \left(\int_0^1 y_{j,t}^{\frac{\mu-1}{\mu}} dj \right)^{\frac{\mu}{\mu-1}} - \int_0^1 p_{j,t} y_{j,t} dj \right\} . \tag{12}$$

Final goods producer II

We have seen that this problem leads to the following demand function for each individual intermediate input:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\mu} Y_t. \tag{13}$$

Moreover, the aggregate price index is:

$$P_t = \left(\int_0^1 p_{j,t}^{1-\mu} dj\right)^{\frac{1}{1-\mu}}. \tag{14}$$

Intermediate goods producers

An intermediate goods producer j produces output according to $y_{j,t} = A_t h_{j,t}$. Its real profits in period t are

$$\pi_{j,t} = \frac{p_{j,t}}{P_t} y_{j,t} - \frac{W_t}{P_t} h_{j,t} \tag{15}$$

$$= \frac{p_{j,t}}{P_t} y_{j,t} - mc_t y_{j,t}, \tag{16}$$

where $mc_t = \frac{W_t}{P_t}\frac{1}{A_t}$ are real marginal costs. Plugging in the demand function, (13), yields

$$\pi_{j,t} = \frac{p_{j,t}}{P_t} \left(\frac{p_{j,t}}{P_t}\right)^{-\mu} Y_t - mc_t \left(\frac{p_{j,t}}{P_t}\right)^{-\mu} Y_t, \tag{17}$$

Calvo-pricing

- Intermediate goods producers face sticky prices.
- Every period, they can set a new price with probability $1 - \lambda$. With probability λ their price is the same as last period.
- This type of pricing friction is called Calvo pricing.
- It implies that with aggregate stochastic shocks, prices are heterogeneous across firms as some have just reset their price while others are stuck with past prices.
- Maybe unrealistically, the probability to reset ones price does not depend on its distance to the optimal price.

Optimal pricing

Consider the problem of an intermediate goods producer which can reset its price in period t. It knows that in period $t+1, t+2, t+s$ it is stuck with the price with probability $\lambda^1, \lambda^2, \lambda^s$. It maximizes the expected real discounted profits resulting from its pricing decision:

$$\max_{p_{j,t}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{C_{t+s}^{-\gamma}}{C_t^{-\gamma}} \lambda^s \left[\frac{p_{j,t}}{P_{t+s}} \left(\frac{p_{j,t}}{P_{t+s}} \right)^{-\mu} Y_{t+s} - mc_{t+s} \left(\frac{p_{j,t}}{P_{t+s}} \right)^{-\mu} Y_{t+s} \right] \right\} \tag{18}$$

with FOC:

$$\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{C_{t+s}^{-\gamma}}{C_t^{-\gamma}} \lambda^s \left[(1-\mu) p_{j,t}^{-\mu} P_{t+s}^{\mu-1} Y_{t+s} + \mu p_{j,t}^{-\mu-1} P_{t+s}^{\mu} mc_{t+s} Y_{t+s} \right] \right\} = 0. \tag{19}$$

Optimal pricing II

Rewriting the optimal pricing decision:

$$(1-\mu)p_{j,t}^{-\mu}\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^s C_{t+s}^{-\gamma}\lambda^s\left[P_{t+s}^{\mu-1}Y_{t+s}\right]\right\}$$
$$+\mu p_{j,t}^{-\mu-1}\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^s C_{t+s}^{-\gamma}\lambda^s\left[P_{t+s}^{\mu}mc_{t+s}Y_{t+s}\right]\right\}=0. \quad (20)$$

Dividing by individual prices yields:

$$(1-\mu)\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^s C_{t+s}^{-\gamma}\lambda^s\left[P_{t+s}^{\mu-1}Y_{t+s}\right]\right\}$$
$$+\mu p_{j,t}^{-1}\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^s C_{t+s}^{-\gamma}\lambda^s\left[P_{t+s}^{\mu}mc_{t+s}Y_{t+s}\right]\right\}=0. \quad (21)$$

Optimal pricing III

Finally, solving for individual prices yields:

$$p_{j,t} = \frac{\mu}{\mu - 1} \frac{\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \lambda^s C_{t+s}^{-\gamma} P_{t+s}^{\mu} mc_{t+s} Y_{t+s} \right\}}{\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \lambda^s C_{t+s}^{-\gamma} P_{t+s}^{\mu-1} Y_{t+s} \right\}} = p_t^{\circ} \tag{22}$$

The optimal price is independent of j, i.e., all producers choose the same price, p_t°, that depends only on aggregate variables. For later, it will be useful to write things recursively. Define:

$$X_{1,t} = C_t^{-\gamma} P_t^{\mu} mc_t Y_t + \beta \lambda \mathbb{E}_t X_{1,t+1} \tag{23}$$

$$X_{2,t} = C_t^{-\gamma} P_t^{\mu-1} Y_t + \beta \lambda \mathbb{E}_t X_{2,t+1}. \tag{24}$$

Optimal pricing IV

Then we have

$$p_t^{\circ} = \frac{\mu}{\mu - 1} \frac{X_{1,t}}{X_{2,t}}. \tag{25}$$

Note, with flexible prices, $\lambda = 0$, we have

$$p_t^{\circ} = \frac{\mu}{\mu - 1} \frac{C_t^{-\gamma} P_t^{\mu} mc_t Y_t}{C_t^{-\gamma} P_t^{\mu - 1} Y_t} = \frac{\mu}{\mu - 1} P_t mc_t, \tag{26}$$

i.e., prices are the static constant mark-up over nominal marginal costs. With sticky prices, mark-ups are different (and possibly time-varying) because producers take into account that they cannot reset their price every period.

Aggregation and stochastic processes

- We are now going to aggregate the individual producer decisions.
- Moreover, we will assume exogenous stochastic processes and a process for taxes.
- This will allow us to simplify the aggregate household's budget constraint.
- Moreover, we will obtain a relationship between aggregate output and price dispersion.

Aggregate profits

Total profits are the aggregate of individual profits:

$$\Pi_t = \int_0^1 \pi_{j,t} dj = \int_0^1 \frac{p_{j,t}}{P_t} y_{j,t} - \frac{W_t}{P_t} h_{j,t} dj. \tag{27}$$

Aggregating labor yields:

$$\Pi_t = \int_0^1 \frac{p_{j,t}}{P_t} y_{j,t} dj - \frac{W_t}{P_t} H_t. \tag{28}$$

Plugging in the individual demand function yields

$$\Pi_t = \int_0^1 \left(\frac{p_{j,t}}{P_t} \right)^{1-\mu} Y_t dj - \frac{W_t}{P_t} H_t. \tag{29}$$

Aggregate profits II

Taking out aggregate terms from the integral yields:

$$\Pi_t = Y_t P_t^{\mu-1} \int_0^1 p_{j,t}^{1-\mu} dj - \frac{W_t}{P_t} H_t. \tag{30}$$

Finally, using equation (14) gives us:

$$\Pi_t = Y_t - \frac{W_t}{P_t} H_t. \tag{31}$$

Firms' aggregate profits are total output minus total real labor income.

Exogenous processes and fiscal policy

We assume productivity follows an $AR(1)$ process in logs:

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1}, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2). \tag{32}$$

Money follows an $AR(1)$ process in its growth rate:

$$\ln M_{t+1} - \ln M_t = (1 - \rho_m)m^* + \rho_m(\ln M_t - \ln M_{t-1}) + \epsilon_t^m. \tag{33}$$

The government runs a balanced budget:

$$T_t = -\frac{M_{t+1} - M_t}{P_t}. \tag{34}$$

The equilibrium budget constraint

In equilibrium, bonds are in zero net supply, $B_t = 0$:

$$C_t + \frac{M_{t+1} - M_t}{P_t} = \frac{W_t}{P_t} H_t + \Pi_t - T_t. \tag{35}$$

Plugging in the equilibrium taxes yields:

$$C_t = \frac{W_t}{P_t} H_t + \Pi_t. \tag{36}$$

Substituting for firms' aggregate profits yields:

$$C_t = Y_t. \tag{37}$$

Aggregate output and price dispersion

Start from idiosyncratic demand:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\mu} Y_t. \tag{38}$$

Plugging in the production function and aggregating yields

$$\int_0^1 A_t h_{j,t} dj = \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\mu} Y_t dj \tag{39}$$

$$A_t H_t = Y_t \psi_t. \tag{40}$$

with

$$\psi_t = \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\mu} dj. \tag{41}$$

Aggregate output and price dispersion II

$$Y_t = \frac{A_t H_t}{\psi_t}. \tag{42}$$

ψ_t is a measure of price dispersion, and one can show that $\psi_t \geq 1$. Sticky prices lead to price dispersion. With heterogeneous p_{jt}, the final goods producer will demand different quantities from the different intermediate goods producers. However, we have seen that (with equally productive intermediate goods producers) output is maximized when all inputs are used in equal quantities. Put differently, price stickiness creates a misallocation that reduces output.

Working with a stationary system

- So far, the model has still money and prices in their levels.
- We know that these variables are not stationary because we allow for a constant money growth.
- We have seen before that the real money supply and the inflation rate are stationary.
- Hence, we are going to rewrite the model in terms of these stationary variables.

The growth rate of real money

We have already seen that we can rewrite

$$\ln M_{t+1} - \ln M_t = (1-\rho_m)m^* + \rho_m(\ln M_t - \ln M_{t-1}) + \epsilon_t^m \tag{43}$$

using the definition $m_t = \frac{M_{t+1}}{P_t}$ as

$$\Delta \ln m_{t+1} + \pi_t = (1-\rho_m)m^* + \rho_m(\Delta \ln m_t + \pi_{t-1}) + \epsilon_t^m. \tag{44}$$

Aggregate price dynamics

Next, consider the aggregate price index (14):

$$P_t^{1-\mu} = \int_0^1 p_{j,t}^{1-\mu} dj. \tag{45}$$

Note that a fraction of producers $1 - \lambda$ resets its price to the optimal price and the remainder keeps its old price:

$$P_t^{1-\mu} = \int_0^{1-\lambda} p_t^{\circ,1-\mu} dj + \int_{1-\lambda}^1 p_{j,t-1}^{1-\mu} dj. \tag{46}$$

Aggregate price dynamics II

Those resetting the price is a random draw from all producers. Hence, the average of their price is simply the aggregate price level the last period:

$$P_t^{1-\mu} = (1-\lambda) p_t^{\circ, 1-\mu} + \lambda P_{t-1}^{1-\mu}. \tag{47}$$

Note that the price level is non-stationary. Hence, we rewrite this in terms of inflation:

$$(1+\pi_t)^{1-\mu} = (1-\lambda)(1+\pi_t^{\circ})^{1-\mu} + \lambda \tag{48}$$

Dynamics of price dispersion

We now turn to the equation for price dispersion (41):

$$\psi_t = \int_0^1 \left(\frac{p_{j,t}}{P_t} \right)^{-\mu} dj \tag{49}$$

Using the fact that $1 - \lambda$ reset to the optimal price, and λ retain their price we have

$$\psi_t = \int_0^{1-\lambda} \left(\frac{p_t^{\circ}}{P_t} \right)^{-\mu} dj + \int_{1-\lambda}^1 \left(\frac{p_{j,t-1}}{P_t} \right)^{-\mu} dj \tag{50}$$

We again want to rewrite this in terms of inflation:

$$\psi_t = \int_0^{1-\lambda} \left(\frac{p_t^{\circ}}{P_{t-1}} \right)^{-\mu} \left(\frac{P_{t-1}}{P_t} \right)^{-\mu} dj + \int_{1-\lambda}^1 \left(\frac{p_{j,t-1}}{P_{t-1}} \right)^{-\mu} \left(\frac{P_{t-1}}{P_t} \right)^{-\mu} dj \tag{51}$$

Dynamics of price dispersion II

Note that

$$\int_{1-\lambda}^{1} \left(\frac{p_{j,t-1}}{P_{t-1}} \right)^{-\mu} dj = \lambda \psi_{t-1} \tag{52}$$

and, hence, we have

$$\psi_t = (1-\lambda)\left(1+\pi_t^{\circ}\right)^{-\mu}(1+\pi_t)^{\mu} + \lambda\psi_{t-1}(1+\pi_t)^{\mu}, \tag{53}$$

which implies that price dispersion is an $AR(1)$ process with auto-regressive coefficient λ.

Dynamics of the optimal price

The optimal price, (25), is also written in terms of its level. Rewriting in growth rates:

$$p_t^\circ = \frac{\mu}{\mu-1}\frac{X_{1,t}}{X_{2,t}} \tag{54}$$

$$= \frac{\mu}{\mu-1} P_t \frac{\frac{X_{1,t}}{P_t^{\mu}}}{\frac{X_{2,t}}{P_t^{\mu-1}}}. \tag{55}$$

Dividing by last period's price level yields

$$(1+\pi_t^\circ) = \frac{\mu}{\mu-1}(1+\pi_t)\frac{\frac{X_{1,t}}{P_t^{\mu}}}{\frac{X_{2,t}}{P_t^{\mu-1}}}. \tag{56}$$

Dynamics of the optimal price II

Now we need to rewrite the terms $X_{1,t}, X_{2,t}$:

$$x_{1,t} = \frac{X_{1,t}}{P_t^{\mu}} = C_t^{-\gamma} mc_t Y_t + \frac{\beta\lambda\mathbb{E}_t X_{1,t+1}}{P_t^{\mu}} \tag{57}$$

$$= C_t^{-\gamma} mc_t Y_t + \beta\lambda\mathbb{E}_t \left\{ \frac{X_{1,t+1}}{P_{t+1}^{\mu}} (1+\pi_{t+1})^{\mu} \right\} \tag{58}$$

$$= C_t^{-\gamma} mc_t Y_t + \beta\lambda\mathbb{E}_t \left\{ x_{1,t+1} (1+\pi_{t+1})^{\mu} \right\}. \tag{59}$$

Similarly, for $X_{2,t}$ we have:

$$x_{2,t} = \frac{X_{2,t}}{P_t^{\mu-1}} = C_t^{-\gamma} Y_t + \beta\lambda\mathbb{E}_t \left\{ x_{2,t+1} (1+\pi_{t+1})^{\mu-1} \right\}. \tag{60}$$

Dynamics of the optimal price III

Putting things together, we have

$$(1 + \pi_t^\circ) = \frac{\mu}{\mu - 1} (1 + \pi_t) \frac{x_{1,t}}{x_{2,t}} \tag{61}$$

with

$$x_{1,t} = C_t^{-\gamma} mc_t Y_t + \beta\lambda\mathbb{E}_t \{x_{1,t+1}(1 + \pi_{t+1})^{\mu}\} \tag{62}$$

$$x_{2,t} = C_t^{-\gamma} Y_t + \beta\lambda\mathbb{E}_t \left\{x_{2,t+1}(1 + \pi_{t+1})^{\mu-1}\right\}. \tag{63}$$

Dynamics of the mark up

Putting things together, we have

$$(1 + \pi_t^{\circ}) = \frac{\mu}{\mu - 1} (1 + \pi_t) \frac{x_{1,t}}{x_{2,t}} \tag{64}$$

Whenever $\pi_t^{\circ} > \pi_t$, some firms charge a mark up below their target because they cannot adjust prices. They stay in the economy as they still charge a positive mark up. In fact, we will see that this is always the case when $\pi_t > \pi^{ss}$, i.e., increasing inflation leads to a decreasing mark up. We have seen that a decreasing mark up increases real wages and incentevizes work.

Equilibrium conditions

Household's FOCs:

$$C_t^{-\gamma} = \beta \mathbb{E}_t \Big\{ C_{t+1}^{-\gamma} (1 + i_t)(1 + \pi_{t+1})^{-1} \Big\} \tag{65}$$

$$m_t = \varphi C_t^{\gamma} \frac{1 + i_t}{i_t} \tag{66}$$

$$\phi H_t^{\eta} = C_t^{-\gamma} \frac{W_t}{P_t}. \tag{67}$$

Budget constraint and output:

$$Y_t = C_t \tag{68}$$

$$Y_t = \frac{A_t H_t}{\psi_t} \tag{69}$$

$$\psi_t = (1 - \lambda) \left(1 + \pi_t^{\circ}\right)^{-\mu} (1 + \pi_t)^{\mu} + \lambda \psi_{t-1} (1 + \pi_t)^{\mu}. \tag{70}$$

Equilibrium conditions II

Firm optimality:

$$(1+\pi_t)^{1-\mu} = (1-\lambda)(1+\pi_t^{\circ})^{1-\mu} + \lambda \tag{71}$$

$$(1+\pi_t^{\circ}) = \frac{\mu}{\mu-1}(1+\pi_t)\frac{x_{1,t}}{x_{2,t}} \tag{72}$$

$$x_{1,t} = C_t^{-\gamma}\frac{W_t}{A_t P_t}Y_t + \beta\lambda\mathbb{E}_t\left\{x_{1,t+1}(1+\pi_{t+1})^{\mu}\right\} \tag{73}$$

$$x_{2,t} = C_t^{-\gamma}Y_t + \beta\lambda\mathbb{E}_t\left\{x_{2,t+1}(1+\pi_{t+1})^{\mu-1}\right\}. \tag{74}$$

Exogenous processes:

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1} \tag{75}$$

$$\Delta \ln m_{t+1} + \pi_t = (1-\rho_m)m^* + \rho_m(\Delta \ln m_t + \pi_{t-1}) + \epsilon_t^m. \tag{76}$$

Steady state

Before turning to a quantitative analysis of the dynamics of the system, it is instructive to study its long-run properties. In steady state, we have

$$A_t = 1 \tag{77}$$

$$\Delta \ln m_{t+1} = m^* \tag{78}$$

$$\pi_t = \pi^{ss} = m^*. \tag{79}$$

From the bond optimality, with $C_t = C_{t+1}$ we have:

$$i^{ss} = \frac{1}{\beta}(1 + \pi^{ss}) - 1. \tag{80}$$

Steady state II

Now solve the price-dynamic equation for target inflation:

$$(1+\pi^{ss})^{1-\mu} = (1-\lambda)(1+\pi^{\circ,ss})^{1-\mu} + \lambda \tag{81}$$

$$\pi^{\circ,ss} = \left(\frac{(1+\pi^{ss})^{1-\mu} - \lambda}{1-\lambda}\right)^{\frac{1}{1-\mu}} - 1 \tag{82}$$

Note that when $\pi^{ss} = 0$, $\pi^{\circ,ss} = \pi^{ss} = 0$. When $\pi^{ss} > 0$, $\pi^{\circ,ss} > \pi^{ss}$, i.e., the targeted inflation rate is higher than the realized inflation rate (because not all producers reset their price).

Steady state III

Now write the dynamics of the price dispersion in steady state:

$$\psi^{ss} = (1-\lambda)\left(1+\pi^{\circ,ss}\right)^{-\mu}\left(1+\pi^{ss}\right)^{\mu} + \lambda\psi^{ss}\left(1+\pi^{ss}\right)^{\mu} \quad (83)$$

$$\psi^{ss}(1-\lambda(1+\pi^{ss})^{\mu}) = (1-\lambda)\left(\frac{1+\pi^{ss}}{1+\pi^{\circ,ss}}\right)^{\mu}. \quad (84)$$

Note that when $\pi^{ss}=0$, $\psi^{ss}=1$. One can show that with any other π^{ss}, $\psi^{ss}>1$, i.e., there is a production inefficiency coming from equilibrium price dispersion.

Steady state IV

From the dynamics of the optimal price we have:

$$\frac{x_1^{ss}}{x_2^{ss}} = \frac{1+\pi^{\circ,ss}}{1+\pi^{ss}} \frac{\mu-1}{\mu}. \tag{85}$$

Next, solve for x_1^{ss} and x_2^{ss}:

$$x_1^{ss} = \frac{mc^{ss}\left(C^{ss}\right)^{-\gamma} Y^{ss}}{1-\beta\lambda(1+\pi^{ss})^{\mu}} \tag{86}$$

$$x_2^{ss} = \frac{\left(C^{ss}\right)^{-\gamma} Y^{ss}}{1-\beta\lambda(1+\pi^{ss})^{\mu-1}}. \tag{87}$$

Steady state V

Which implies:

$$\frac{x_1^{ss}}{x_2^{ss}} = mc^{ss}\frac{1-\beta\lambda(1+\pi^{ss})^{\mu-1}}{1-\beta\lambda(1+\pi^{ss})^{\mu}}. \tag{88}$$

Putting things together gives

$$mc^{ss} = \frac{1-\beta\lambda(1+\pi^{ss})^{\mu}}{1-\beta\lambda(1+\pi^{ss})^{\mu-1}}\frac{1+\pi^{\circ,ss}}{1+\pi^{ss}}\frac{\mu-1}{\mu}. \tag{89}$$

When $\pi^{ss}=0$, we have $\frac{W}{P}=\frac{\mu-1}{\mu}A$. One can show that for any other π^{ss}, $\frac{W}{P}<\frac{\mu-1}{\mu}A$, i.e., producers set higher mark-ups.

Steady state VI

Now use the labor supply condition with $A^{ss} = 1$:

$$\phi \left(H^{ss}\right)^{\eta} = \left(Y^{ss}\right)^{-\gamma} mc^{ss} \tag{90}$$

From the production function we have $Y^{ss} = \frac{H^{ss}}{\psi^{ss}}$ and, hence,

$$H^{ss} = \left(\frac{mc^{ss}}{\phi} \left(\psi^{ss}\right)^{\gamma}\right)^{\frac{1}{\eta+\gamma}}. \tag{91}$$

Finally, steady state money demand reads:

$$m^{ss} = \varphi \left(\frac{H^{ss}}{\psi^{ss}}\right)^{\gamma} \frac{1 + i^{ss}}{i^{ss}}. \tag{92}$$

The optimal long-run inflation rate

- We now have all results to determine the long run optimal inflation rate.
- $\pi^{ss} = 0$ insures that there is no price dispersion, $\psi^{ss} = 1$.
- This policy maximizes steady state output.
- Further, we know that $i^{ss} = 0$ maximizes real money holdings.
- This requires $\pi^{ss} = \beta - 1$.

$\Rightarrow$ Optimal long-run inflation must be $\beta - 1 \leq \pi^{ss} \leq 0$.

▸ The cause of inflation

Flexible price equilibrium

Before turning to a quantitative analysis of the dynamics of the system, it is instructive to also study the flexible price equilibrium $\lambda = 0$. Because capital is not a state variable, we can solve this closed-form. We know:

$$\psi_t = 1 \tag{93}$$

$$\frac{W_t}{P_t} = \frac{\mu - 1}{\mu} A_t \tag{94}$$

Flexible price equilibrium II

Plug the real wage into the labor supply decision together with $C_t = Y_t = A_t H_t$ yields:

$$\phi H_t^\eta = (A_t H_t)^{-\gamma} \frac{\mu - 1}{\mu} A_t \tag{95}$$

$$H_t = \left(\frac{1}{\phi} \frac{\mu - 1}{\mu} A_t^{1-\gamma} \right)^{\frac{1}{\gamma+\eta}} . \tag{96}$$

Note, with log utility, H_t would be a constant. Without capital, an increase in productivity does not provide incentives to work harder and accumulate more capital. This also implies, that in the New Keynesian Model, procyclicality of hours worked must result from the volatility in price dispersion and mark-ups.

Flexible price equilibrium III

Finally, using the production function yields:

$$Y_t = \left(\frac{1}{\phi} \frac{\mu - 1}{\mu} \right)^{\frac{1}{\gamma+\eta}} A_t^{\frac{1+\eta}{\gamma+\eta}}. \tag{97}$$

Note, with flexible prices, the classical dichotomy holds. Output does not depend on any nominal variables and fluctuates proportional with productivity. To make this explicit, consider the log-linearization:

$$Y^{ss}(1 + \hat{Y}_t) = \left(\frac{1}{\phi} \frac{\mu - 1}{\mu} \right)^{\frac{1}{\gamma+\eta}} A^{ss} \left(1 + \frac{1+\eta}{\gamma+\eta} \hat{A}_t \right) \tag{98}$$

$$\hat{Y}_t = \frac{1+\eta}{\gamma+\eta} \hat{A}_t. \tag{99}$$

Log-linearization

We now return to the analysis of the dynamic model with sticky prices. The final step before the quantitative analysis will be to look at the log-linearized model. It will be mathematically convenient to approximate around the zero inflation steady state, such that

$$\pi^{ss} = 0$$
$$i^{ss} = \frac{1}{\beta} - 1$$
$$\psi^{ss} = 1.$$

Log-linearization: Budget constraint

We start with the budget constraint:

$$Y_t = C_t \tag{100}$$

$$\hat{Y}_t = \hat{C}_t. \tag{101}$$

The equation looks trivial but it has some deep implications. The New Keynesian model is what we call demand-driven. Whatever households are demanding in terms of consumption, firms will produce.

Log-linearization: Bond equation

Now consider the optimality equation for bonds and substitute $Y_t = C_t$:

$$Y_t^{-\gamma} = \beta \mathbb{E}_t \Big\{ Y_{t+1}^{-\gamma} (1 + i_t)(1 + \pi_{t+1})^{-1} \Big\}. \tag{102}$$

Now substitute for Y_{t+s} its first-order approximations (but not for the rates):

$$(1 - \gamma \hat{Y}_t) = \beta \mathbb{E}_t \Big\{ (1 - \gamma \hat{Y}_{t+1})(1 + i_t)(1 + \pi_{t+1})^{-1} \Big\} \tag{103}$$

Log-linearization: Bond equation II

Take logs and rearrange:

$$\mathbb{E}_t \hat{Y}_{t+1} - \hat{Y}_t = \frac{1}{\gamma} \left[(i_t + \ln(\beta)) - \mathbb{E}_t \pi_{t+1} \right]. \tag{104}$$

Note, with $\beta \approx 1$, $\ln(\beta) \approx -i^{ss}$. Hence, the rates are in deviations from their steady states. This equation is key. It says that the deviation of expected output growth from its log steady state depends only on the expected deviation of the real interest rate from its steady state. When the real interest rate is high, output growth is high, i.e., output today is low.

Log-linearization: Bond equation III

$$\mathbb{E}_t \hat{Y}_{t+1} - \hat{Y}_t = \frac{1}{\gamma} \left[(i_t + \ln(\beta)) - \mathbb{E}_t \pi_{t+1} \right]. \tag{105}$$

One way to think about this equation is as a demand equation. A high real interest rate leads households to defer consumption from today to tomorrow. As $Y = C$, this leads to low output today.

Log-linearization: Bond equation IV

$$\mathbb{E}_t \hat{Y}_{t+1} - \hat{Y}_t = \frac{1}{\gamma}\left[(i_t + \ln(\beta)) - \mathbb{E}_t \pi_{t+1}\right]. \tag{106}$$

This equation is sometimes called the New Keynesian IS-curve because it says that output decreases in the real interest rate. Note, however, that the mechanism is completely different from the classical Keynesian model.

CKM: $r_t \uparrow \Rightarrow I_t \downarrow \Rightarrow Y_t \downarrow$.

NKM: It is all about intertemporal substitution.

Log-linearization: Money demand

$$m_t = \varphi C_t^{\gamma} \frac{1 + i_t}{i_t} \tag{107}$$

Now substitute the approximations for real money and output:

$$m^{ss}(1 + \hat{m}_t) = \varphi \left(Y^{ss}\right)^{\gamma} (1 + \gamma \hat{Y}_t) \frac{1 + i_t}{i_t} \tag{108}$$

Now divide by m^{ss}:

$$(1 + \hat{m}_t) = (1 + \gamma \hat{Y}_t) \frac{i^{ss}}{1 + i^{ss}} \frac{1 + i_t}{i_t}. \tag{109}$$

Log-linearization: Money demand II

Now, take logs:

$$\hat{m}_t = \gamma \hat{Y}_t + (i_t - i^{ss}) - (\ln(i_t) - \ln(i^{ss})). \tag{110}$$

Finally, note the first order Taylor series expansion of $\ln(i_t)$ around its steady state is

$$\ln(i_t) \approx \ln(i^{ss}) + \frac{1}{i^{ss}}(i_t - i^{ss}) \tag{111}$$

$$\hat{m}_t = \gamma \hat{Y}_t + (i_t - i^{ss}) - (\frac{1}{i^{ss}}(i_t - i^{ss})) \tag{112}$$

$$\hat{m}_t = \gamma \hat{Y}_t + [1 - \frac{\beta}{1-\beta}](i_t - i^{ss}). \tag{113}$$

Log-linearization: Money demand III

$$\hat{m}_t = \gamma \hat{Y}_t + [1 - \frac{\beta}{1-\beta}](i_t - i^{ss}). \quad (114)$$

The deviation of money demand from its log steady state depends positively on output deviations and negatively on deviations of the interest rate.

This is as the LM-curve in the classical Keynesian model. For a fixed money supply, equilibrium in the money market implies that there is a positive relationship between output deviations and the interest rate deviations (when $\beta > 0.5$).

Log-linearization: Inflation dynamics

$$(1+\pi_t)^{1-\mu} = (1-\lambda)(1+\pi_t^\circ)^{1-\mu} + \lambda \tag{115}$$

It is convenient to define two auxilary variables, $V_t^1 = 1+\pi_t^\circ$ and $V_t^2 = 1+\pi_t$. Note, that both have a steady state of 1 at the zero inflation steady state, and $\ln V_t^1 \approx \pi_t^\circ$, and $\ln V_t^2 \approx \pi_t$:

$$(1+(1-\mu)\hat{V}_t^1) = (1-\lambda)(1+(1-\mu)\hat{V}_t^2) + \lambda \tag{116}$$

$$(1+(1-\mu)\ln V_t^1) = (1-\lambda)(1+(1-\mu)\ln V_t^2) + \lambda \tag{117}$$

$$(1+(1-\mu)\pi_t) = (1-\lambda)(1+(1-\mu)\pi_t^\circ) + \lambda \tag{118}$$

$$\pi_t = (1-\lambda)\pi_t^\circ. \tag{119}$$

This result is obvious, inflation deviations are a weighted average of deviations of targeted inflation and deviations of inflation of those not adjusting prices. The latter is obviously zero.

Log-linearization: Price dispersion dynamics

$$\psi_t = (1-\lambda)\left(1+\pi_t^{\circ}\right)^{-\mu}\left(1+\pi_t\right)^{\mu} + \lambda\psi_{t-1}\left(1+\pi_t\right)^{\mu}. \tag{120}$$

$$(1+\hat{\psi}_t) = (1-\lambda)\left(1-\mu\pi_t^{\circ}\right)\left(1+\mu\pi_t\right) + \lambda(1+\mu\pi_t+\hat{\psi}_{t-1}). \tag{121}$$

Multiplying out and dropping higher-order terms yields:

$$\hat{\psi}_t = \lambda\hat{\psi}_{t-1} + \mu\pi_t - \mu(1-\lambda)\pi_t^{\circ}. \tag{122}$$

Log-linearization: Price dispersion dynamics II

Plugging in $\pi_t = (1-\lambda)\pi_t^{\circ}$ from the inflation dynamics yields

$$\hat{\psi}_t = \lambda \hat{\psi}_{t-1}. \tag{123}$$

Hence, when we start from the zero inflation steady state with $\psi_t = 1$, we will have $\psi_{t+s} = 1$ for all future periods s. Put differently, with a first-order approximation, price dispersion is unimportant for the dynamics of the New Keynesian model. For it to matter, we would have to look at higher order approximations.

Log-linearization: Labor supply

I use $C_t = Y_t$ from the budget constraint and define $w_t = \frac{W_t}{P_t}$:

$$\phi H_t^{\eta} = Y_t^{-\gamma} w_t \tag{124}$$

$$H^{ss}(1 + \eta \hat{H}_t) = \frac{1}{\phi} \left(Y^{ss}\right)^{-\gamma} w^{ss}(1 - \gamma \hat{Y}_t + \hat{w}_t) \tag{125}$$

$$\hat{H}_t = \frac{1}{\eta} \left[\hat{w}_t - \gamma \hat{Y}_t\right]. \tag{126}$$

As before, wages rise with real wages and decrease when consumption (output) increases because of a wealth effect. The strength of the response depends on the Fisher labor supply elasticity, $\frac{1}{\eta}$.

Log-linearization: Production function

$$Y_t = \frac{A_t H_t}{\psi_t} \tag{127}$$

$$Y^{ss}(1 + \hat{Y}_t) = \frac{H^{ss}}{\psi^{ss}}(1 + \hat{A}_t + \hat{H}_t - \hat{\psi}_t) \tag{128}$$

$$\hat{Y}_t = \hat{A}_t + \hat{H}_t, \tag{129}$$

Now substitute the optimal labor response (126):

$$\hat{Y}_t = \hat{A}_t + \frac{1}{\eta}\left[\hat{w}_t - \gamma \hat{Y}_t\right] \tag{130}$$

$$\eta(\hat{Y}_t - \hat{A}_t) = \hat{w}_t - \gamma \hat{Y}_t. \tag{131}$$

Log-linearization: Production function II

Now use $\hat{w}_t = \hat{m}c_t + \hat{A}_t$

$$\eta(\hat{Y}_t - \hat{A}_t) = \hat{m}c_t + \hat{A}_t - \gamma \hat{Y}_t \quad (132)$$

$$\hat{m}c_t = (\eta + \gamma)\hat{Y}_t - (1 + \eta)\hat{A}_t. \quad (133)$$

We now use the trick that we have shown above that in the flexible price equilibrium $\hat{A}_t = \frac{\gamma+\eta}{1+\eta}\hat{Y}_t^f$:

$$\hat{m}c_t = (\eta + \gamma)[\hat{Y}_t - \hat{Y}_t^f], \quad (134)$$

where $\hat{Y}_t - \hat{Y}_t^f$ is the deviation of output from the flexible price equilibrium which we will call the output gap.

Log-linearization: Production function III

$$\hat{mc}_t = (\eta + \gamma)[\hat{Y}_t - \hat{Y}_t^f], \tag{135}$$

- Real marginal costs are above steady state when output is above the flexible price equilibrium.
- Recall from the flexible price equilibrium that the real marginal costs are simply the inverse price mark-up, $\frac{\mu-1}{\mu}$.
- Hence, when output is above the flexible price equilibrium, mark-ups are below the desired mark-ups. As seen before, a decrease in mark-ups incentivizes working and, hence, increases output above the flexible price equilibrium.

Log-linearization: Meaning of the output gap

$$\hat{mc}_t = (\eta + \gamma)[\hat{Y}_t - \hat{Y}_t^f], \tag{136}$$

- Why is the gap from actual to the flexible price equilibrium an interesting measure?
- One needs to differentiate between the level effect and the fluctuation effect resulting from imperfect competition.
- Level effect: On average, output is lower than under perfect competition. There is nothing we can do about that using business cycle policies.
- Fluctuation effect: Sticky prices create inefficient business cycle variations to the economy. We will see that this is something we can do something about.

Combine (126) and (129)

$$\hat{H}_t = \frac{1}{\gamma + \eta}[\hat{w}_t - \gamma \hat{A}_t] \tag{137}$$

To know what happens to labor supply, we only need to know whether productivity changes by more than the real wage.

Log-linearization: Inflation target dynamics

We start with log-linearizing the auxilary variable $x_{1,t}$:

$$x_{1,t} = C_t^{-\gamma} \frac{W_t}{A_t P_t} Y_t + \beta\lambda\mathbb{E}_t \left\{x_{1,t+1}(1+\pi_{t+1})^{\mu}\right\} \tag{138}$$

$$x_1^{ss}(1+\hat{x}_{1,t}) = (Y^{ss})^{1-\gamma} mc^{ss}[1+(1-\gamma)\hat{Y}_t + \hat{m}c_t] \\ + \beta\lambda x_1^{ss}\mathbb{E}_t(1+\hat{x}_{1,t+1}+\mu\pi_{t+1}) \tag{139}$$

Now, using (86) yields:

$$\hat{x}_{1,t} = (1-\beta\lambda)[(1-\gamma)\hat{Y}_t + \hat{m}c_t] + \beta\lambda\mathbb{E}_t(\hat{x}_{1,t+1}+\mu\pi_{t+1}). \tag{140}$$

Similarly, we have for $x_{2,t}$:

$$\hat{x}_{2,t} = (1-\beta\lambda)(1-\gamma)\hat{Y}_t + \beta\lambda\mathbb{E}_t(\hat{x}_{2,t+1}+(\mu-1)\pi_{t+1}). \tag{141}$$

Log-linearization: Inflation target dynamics II

Now consider the inflation target dynamics:

$$(1+\pi_t^\circ) = \frac{\mu}{\mu-1}(1+\pi_t)\frac{x_{1,t}}{x_{2,t}} \tag{142}$$

$$(1+\pi_t^\circ) = \frac{\mu}{\mu-1}\frac{x_1^{ss}}{x_2^{ss}}(1+\pi_t+\hat{x}_{1,t}-\hat{x}_{2,t}) \tag{143}$$

Using (85) this simplifies to:

$$\pi_t^\circ = \pi_t + \hat{x}_{1,t} - \hat{x}_{2,t}. \tag{144}$$

Using (119) gives

$$\frac{\lambda}{1-\lambda}\pi_t = \hat{x}_{1,t} - \hat{x}_{2,t}. \tag{145}$$

Log-linearization: Inflation target dynamics III

Now using the above results,

$$\hat{x}_{1,t} - \hat{x}_{2,t} = (1-\beta\lambda)\hat{mc}_t + \beta\lambda\mathbb{E}_t(\hat{x}_{1,t+1} - \hat{x}_{2,t+1} + \pi_{t+1}). \tag{146}$$

Combining things, we have:

$$\frac{\lambda}{1-\lambda}\pi_t = (1-\beta\lambda)\hat{mc}_t + \beta\lambda\mathbb{E}_t(\frac{\lambda}{1-\lambda}\pi_{t+1} + \pi_{t+1}) \tag{147}$$

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\hat{mc}_t + \beta\mathbb{E}_t\pi_{t+1} \tag{148}$$

Log-linearization: Inflation target dynamics IV

Using (134), we can write this as

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta+\gamma)[\hat{Y}_t - \hat{Y}_t^f] + \beta \mathbb{E}_t \pi_{t+1} \tag{149}$$

- The equation is usually called the New Keynesian Phillips-curve.
- It relates inflation dynamics to the output gap.
- Note, inflation dynamics are forward-looking and not, as in the classical Phillips curve, backward-looking.
- Moreover, it relates the dynamics to the output gap and not the unemployment rate.

Log-linearization: Inflation target dynamics V

Assume that expected long-run inflation is at its steady state, i.e., $\mathbb{E}_t\pi_{t+\infty} = 0$. Then we can write the New Keynesian Phillips curve as

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s\hat{m}c_{t+s} \tag{150}$$

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta+\gamma)\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s[\hat{Y}_t - \hat{Y}_t^f] \tag{151}$$

- Current inflation depends on expectations of future real marginal costs which are the inverse mark-ups.
- If expected future mark-ups are low, firms that can adjust their price will choose a high price (high inflation) today.
- Similarly, current inflation depends on future output gaps.

Log-linearization: Inflation target dynamics VI

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{mc}_{t+s} \tag{152}$$

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta+\gamma)\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s [\hat{Y}_t - \hat{Y}_t^f] \tag{153}$$

- Note, in RBC, inflation mainly results from aligning prices with a current fixed levels of output and a stochastic current money supply.
- In the NKM, inflation is all about firms setting prices given expectations over future states of the world.

Inflation target dynamics and forward guidance

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{m}c_{t+s} \tag{154}$$

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta+\gamma)\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s [\hat{Y}_t - \hat{Y}_t^f] \tag{155}$$

- This insight lies at the heart of what central banks call "forward-guidance".
- If the central bank can promise a positive output gap in the future by having inflation above steady state in the future, it will generate inflation today.
- After the financial crisis, the ECB and FED promised low future interest rates irrespective of future inflation rates, i.e., they have promised to be "irresponsible".

The debate about inflation in 2022

$$m_t = \varphi C_t^{\gamma} \frac{1 + i_t}{i_t} \tag{156}$$

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta + \gamma)[\hat{Y}_t - \hat{Y}_t^f] + \beta \mathbb{E}_t \pi_{t+1} \tag{157}$$

- Over the last decade, central banks have greatly increased the money supply.
- Those worried about inflation looked at the money demand equation and note that also in the NKM money is neutral in the long-run, i.e., prices need to increase one-to-one with the money supply.
- Those not worried looked at the NKM-Phillips curve and note that the output gap is small and that inflation expectations are "anchored".

The debate about inflation in 2022 II

"Policymakers and analysts generally believe that, as long as longer-term inflation expectations remain anchored, policy can and should look through temporary swings in inflation. Our monetary policy framework emphasizes that anchoring longer-term expectations at 2 percent is important for both maximum employment and price stability. We carefully monitor a wide range of indicators of longer-term inflation expectations."
Jerome Powell, August 2021

- The question is why the output gap remained small and inflation expectations remained low.
- Either people believed that the increase in the money supply was transitory and the central bank would reduce the money supply in the future.
- Or we have the wrong model of inflation. For example, it could be that money demand had also increased.

Calibration

- We are going to use the same calibration as before with an inverse labor supply elasticity $\eta = 0.5$ This leads to $\phi = 12$ to match $H^{ss} = 0.33$.

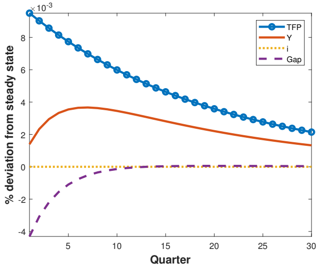
- We will look at the non-inflationary steady-state, $m^* = 0$.

- I set $\rho_m = 0$ to get close to an autocorrelation of HP-filtered inflation of 0.44 and match its standard deviation of (0.006) with $\sigma_m = 0.016$. Hence, a shock is a one-time permanent change in M_t.

- The parameter of the utility of money, φ, matter for the level of money demand which is unimportant for its dynamics. We will set $\varphi = 1$.

- In the data, the average price duration is 13 months in Europe. In the model, average duration in quarters is $\frac{1}{1-\lambda}$. Hence, $\lambda = 0.75$ is reasonable.

Output response to a productivity shock



- Output increases after a positive productivity shock.
- It increases by less than in the flexible price equilibrium.

Output response to a productivity shock II

To understand the mechanism, consider the linearized money demand equation:

$$\hat{Y}_t = \frac{1}{\gamma}\Big[\hat{m}_t - [1 - \frac{\beta}{1-\beta}](i_t - i^{ss})\Big] \tag{158}$$

For output to rise, real money supply needs to rise or the interest rate needs to rise. However, we have just seen that the nominal interest rate remains flat, i.e., the LM-curve is vertical. To see why, use the household's FOCs (5) and (6). We can write the latter as:

$$\frac{\lambda_t}{P_t} = \frac{\varphi}{M_{t+1}} + \beta\frac{\lambda_{t+1}}{P_{t+1}} \tag{159}$$

$$\frac{\lambda_t}{P_t} = \sum_{s=0}^{\infty}\beta^s\frac{\varphi}{M_{t+1+s}} \tag{160}$$

Output response to a productivity shock III

$$\frac{\lambda_t}{P_t} = \sum_{s=0}^{\infty} \beta^s \frac{\varphi}{M_{t+1+s}} \tag{161}$$

However, M_{t+1+s} is exogenous and, hence, does not respond to the shock. Therefore, $\frac{\lambda_t}{P_t}$ and $\frac{\lambda_{t+1}}{P_{t+1}}$ must also remain constant. However, from (5):

$$\frac{\lambda_t}{P_t} = \mathbb{E}_t \Big\{ \beta \frac{\lambda_{t+1}}{P_{t+1}} (1 + i_t) \Big\} \tag{162}$$

it follows that i_t must remain constant.

Output response to a productivity shock IV

Going back to the money demand equation, we have

$$\hat{Y}_t = \frac{1}{\gamma}\hat{m}_t. \tag{163}$$

- In the flexible price equilibrium, the decrease in real marginal costs (increase in supply) would put downward pressure on prices that would be sufficient to equate real money demand with the flexible price output level.
- This cannot happen in the NKM because of sticky prices. Instead, prices adjust only partially leading output to rise less than in the flexible price equilibrium.

Output response to a productivity shock V

With a constant nominal interest rate and deflation we have a positive real interest rate. As a result, we know from the NK IS-curve,

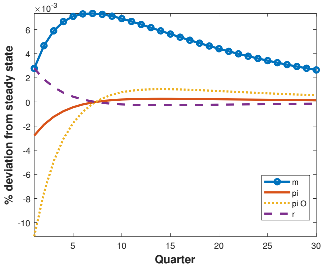
$$\mathbb{E}_t \hat{Y}_{t+1} - \hat{Y}_t = \frac{1}{\gamma}\left[(i_t + \ln(\beta)) - \mathbb{E}_t \pi_{t+1}\right], \tag{164}$$

that output must be increasing initially.

Output response to a productivity shock VI

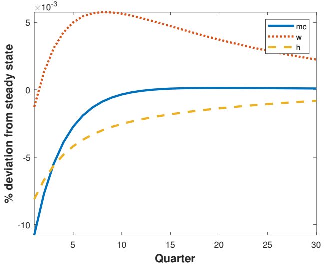
- The discussion implies that there is a role for the central bank to align the economy with its flexible price equilibrium.
- We know that the flexible price equilibrium is efficient, i.e., the central bank can overcome an inefficiency.
- It would need to increase the nominal money supply such that the real money supply increases sufficiently.
- The same argument holds after a negative productivity shock. Output does not decrease sufficiently because prices do not rise quickly enough. Hence, the central bank needs to decrease the money supply.

Real interest rate response to a productivity shock



- As just discussed, the real money supply rises because inflation falls.
- Targeted inflation falls by more than realized inflation because of the price stickiness.
- Different from the RBC model, we get persistent inflation responses because firms can decrease their prices only over time.
- The real interest rate rises.

Hours response to a productivity shock



- Real marginal costs, $\frac{W_t}{P_t A_t}$ decrease because sticky prices imply that the mark up increases.
- Real wages increase by less than labor productivity.
- Hours worked fall.

Hours response to a productivity shock II

Another way to see the response of real marginal costs is to consider the output gap. As the output gap is negative, we know that real marginal costs must respond negatively:

$$\hat{mc}_t = (\eta + \gamma)[\hat{Y}_t - \hat{Y}_t^f]. \tag{165}$$

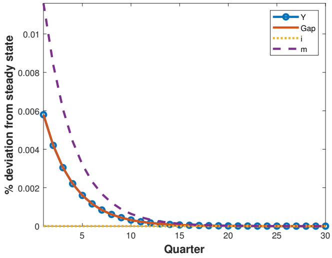
Hence, we know that real wages increase by less than labor productivity:

$$\hat{w}_t = \hat{mc}_t + \hat{A}_t. \tag{166}$$

With wages rising less than labor productivity, it must be that hours worked fall:

$$\hat{H}_t = \frac{1}{\gamma + \eta}[\hat{w}_t - \gamma \hat{A}_t]. \tag{167}$$

Output response to a money shock



- The nominal interest rate, again, does not respond to a positive shock to the growth rate of money. This would be slightly different with $\rho_m > 0$.
- Real money increases.
- Output increases.
- In the flexible price equilibrium, output is unchanged, i.e., all output volatility is inefficient.

Output response to a money shock II

In the flexible price equilibrium, prices would rise to leave real money supply unchanged. As prices are sticky, real money supply increases. From the linearized money demand equation:

$$\hat{Y}_t = \frac{1}{\gamma}\Big[\hat{m}_t - [1 - \frac{\beta}{1-\beta}](i_t - i^{ss})\Big], \tag{168}$$

this increases output.

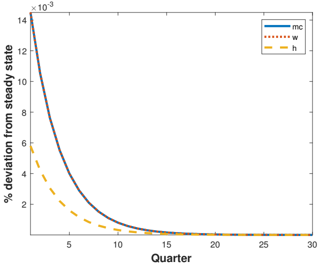
What lies behind the output increase?

Go back to the optimality condition for money holdings:

$$\varphi m^{-1} = C_t^{-\gamma} \frac{i_t}{1 + i_t}. \tag{169}$$

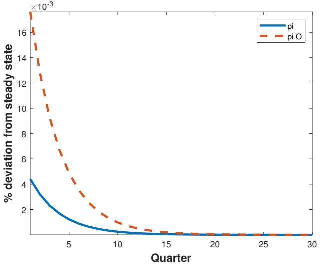
When the real money supply increases, holding additional money for consumption tomorrow loses value, leading the household to consume today.

Hours response to money shock



- As many firms have their current price below the long-run target, average mark-ups decrease and, hence, real marginal costs increase.
- This implies real wages increase. Labor productivity is by definition unchanged.
- Hence, from (137), it must be that hour worked rise.

Inflation response to a money shock



- A positive shock to the growth rate of money increases inflation.
- It increases targeted inflation.

Inflation response to a money shock

We know the output gap is positive, hence, from the NK Phillips curve, we know that inflation increases:

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta+\gamma)[\hat{Y}_t - \hat{Y}_t^f] + \beta\mathbb{E}_t\pi_{t+1} \tag{170}$$

Because of sticky prices, we know that targeted inflation is even higher:

$$\pi_t = (1-\lambda)\pi_t^\circ. \tag{171}$$

Results I

	Y	C	H	TFP	w	i	π
				Data			
Std. %	1.61	1.25	1.9	1.25	0.96	1.07	0.6
				RBC $\eta = 0.5$			
Std. %	1.56	0.45	0.52	1.24	1.10	0.0	0.06
				NKM $\eta = 0.5$			
Std. %	0.79	0.79	1.16	1.24	1.81	0.0	0.6

Correlations

	Y	C	H	TFP	w	π
Data	1					
RBC	1					
NKM	1					
	0.78	1				
C	0.94	1				
	1	1				
	0.87	0.69	1			
H	0.92	0.74	1			
	0.24	0.24	1			
	0.79	0.71	0.49	1		
TFP	1	0.94	0.93	1		
	0.41	0.41	-0.78	1		
	0.12	0.29	-0.06	0.34	1	
w	0.98	0.99	0.84	0.98	1	
	0.95	0.95	0.53	0.11	1	
	0.28	0.37	0.23	0.25	0.32	1
π	-0.34	-0.23	-0.42	-0.34	-0.29	1
	0.65	0.65	0.89	-0.42	0.86	1

Discussion

Improvements:

- Without capital, consumption becomes more volatile.
- Hours are more volatile.
- The procyclicalilty of inflation is the single most improvement and direct evidence for nominal shocks affecting output.
- Wages and TFP are only weakly correlated and the correlation between wages and hours becomes weaker.

Discussion II

Deterioration:

- Without capital, consumption is as volatile as output.
- Hours are now more volatile than output.
- Inflation is too procyclical and it remains negatively correlated with TFP.
- TFP and hours are now negatively correlated. This suggests that the weak correlation between wages and TFP/hours is not by the right mechanism.

Appendix

A little detour: Causes of inflation

Consider a slightly extended version of our model with government spending, G_t, and nominal government debt, D_t, paying a nominal interest rate, i^G_{t-1}. To simplify the notation, I will change somewhat the timing assumption from the main model. The (real) government's budget constrained reads

$$T_t + \frac{M_t - M_{t-1}}{P_t} + \frac{D_t}{P_t} = G_t + (1 + i^G_{t-1})\frac{D_{t-1}}{P_t}. \tag{172}$$

Taxes plus seigniorage plus new debt must equal spending plus the payment on old debt. Shifting one period ahead and rearranging:

$$\frac{D_t}{P_{t+1}} = \frac{1}{1 + i^G_t}\left[\frac{D_{t+1}}{P_{t+1}} + T_{t+1} - G_{t+1} + \frac{M_{t+1} - M_t}{P_{t+1}}\right]. \tag{173}$$

A little detour: Causes of inflation II

$$\frac{D_t}{P_{t+1}} = \frac{1}{1+i_t^G}\left[\frac{D_{t+1}}{P_{t+1}} + T_{t+1} - G_{t+1} + \frac{M_{t+1}-M_t}{P_{t+1}}\right] \quad (174)$$

$$\frac{D_t}{P_t} = \frac{1}{1+i_t^G}\frac{P_{t+1}}{P_t}\left[\frac{D_{t+1}}{P_{t+1}} + T_{t+1} - G_{t+1} + \frac{M_{t+1}-M_t}{P_{t+1}}\right] \quad (175)$$

$$\frac{D_t}{P_t} = \frac{1}{1+r}\left[\frac{D_{t+1}}{P_{t+1}} + T_{t+1} - G_{t+1} + \frac{M_{t+1}-M_t}{P_{t+1}}\right], \quad (176)$$

where r is the real interest rate which, for simplicity, I assume to be constant. Now successively substitute for $\frac{D_{t+s}}{P_{t+s}}$ to iterate this to $S=\infty$ periods forward:

$$\frac{D_t}{P_t} = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^s\left[T_{t+1+s} - G_{t+1+s} + \frac{M_{t+1+s}-M_{t+s}}{P_{t+1+s}}\right] + \lim_{S=\infty}\left\{\left(\frac{1}{1+r}\right)^S\frac{D_{t+1+S}}{P_{t+1+S}}\right\}. \quad (177)$$

A little detour: Causes of inflation III

Now assume real debt does not not explode, i.e., grow faster than the discount rate:

$$\frac{D_t}{P_t} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^s \left[T_{t+1+s} - G_{t+1+s} + \frac{M_{t+1+s} - M_{t+s}}{P_{t+1+s}} \right]. \tag{178}$$

The real value of debt today is the discounted sum of future real government surpluses. Surpluses may either arise because taxes exceed spending or from seigniorage.

Monetary dominance

$$\frac{D_t}{P_t} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^s \left[T_{t+1+s} - G_{t+1+s} + \frac{M_{t+1+s} - M_{t+s}}{P_{t+1+s}} \right]. \qquad (179)$$

In our model, $\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^s \left[\frac{M_{t+1+s} - M_{t+s}}{P_{t+1+s}} \right]$ may take any path and is only determined by the central bank. Fiscal policy is assumed to assure that the level of today's real debt can be serviced by future government surpluses. In fact, in our model without debt, taxes need to adjust every period. This is sometimes referred to a regime of monetary dominance. Under this view, inflation is "always a monetary phenomenon".

Fiscal dominance

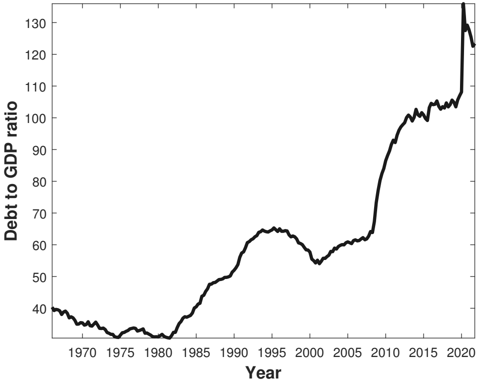
$$\frac{D_t}{P_t} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^s \left[T_{t+1+s} - G_{t+1+s} + \frac{M_{t+1+s} - M_{t+s}}{P_{t+1+s}}\right]. \tag{180}$$

One can also imagine a world in which the fiscal authority is irresponsible and always spends more than it taxes. In that case, the central bank has to make sure that the value of today's real debt can be serviced by future money printing, i.e., under this view, inflation is "always a fiscal phenomenon". This regime is referred to as fiscal dominance.

What regime do we have?

- Proponents of monetary dominance stress the independence of central banks (Biden on independence).
- In the US, price stability and maximum employment are the only objectives of the central bank.
- In Europe, the ECB has price stability as its only objective.
- The European charter explicitly says that the ECB must not conduct monetary policy to bail-out governments.

What regime do we have? The US



- It is hard to believe that the FED will not intervene when the US confronts a default.
- Particularly, as congress can take away its independence at any point.

What regime do we have? Europe

- The independence of the ECB is very hard to alter.
- The ECB was willing to let Greece go bankrupt.
- But shortly thereafter, when it came to Italy and Spain, Mario Draghi announced the doctrine "whatever it takes".
- According to that doctrine, the ECB will do anything within its mandate to preserve the Euro.
- However, a country exciting the Euro and a country's solvency are closely linked.
- Now, the new game in town is avoiding "fragmentation". A not yet very clear concept.

Back

[1] Lawrence J Christiano, Martin Eichenbaum, and Charles L Evans. "Monetary policy shocks: What have we learned and to what end?" In: *Handbook of macroeconomics* 1 (1999), pp. 65–148.

[2] Luis J Alvarez et al. "Sticky prices in the euro area: a summary of new micro-evidence". In: *Journal of the European Economic association* 4.2-3 (2006), pp. 575–584.